QUESTION SIX (15 marks) Use a separate writing booklet. **Marks**

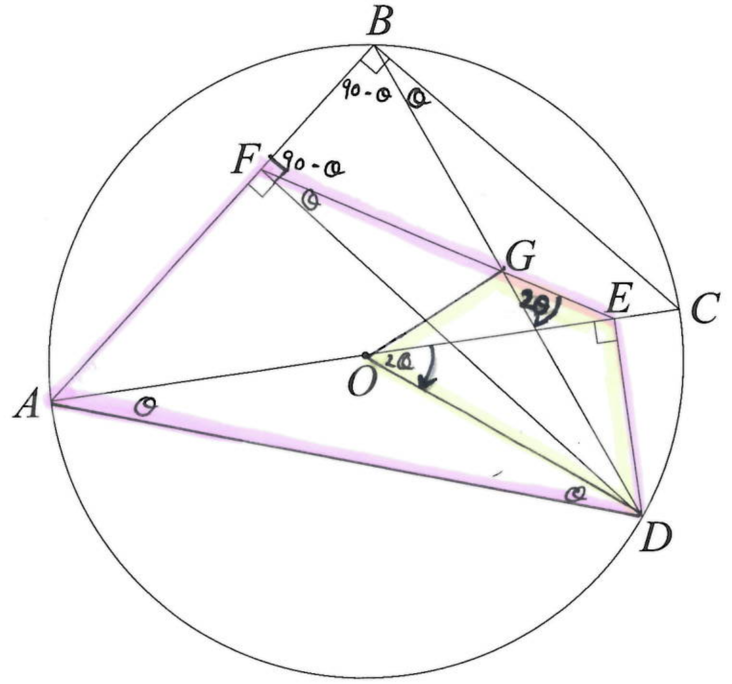
(a) Let $I_n = \displaystyle\int_0^{\frac{\pi}{2}} \sin^n \theta \, d\theta$.

(i) Use integration by parts to show that $I_n = (n-1)\displaystyle\int_0^{\frac{\pi}{2}} \sin^{n-2}\theta \cos^2\theta \, d\theta$. **2**

(ii) Hence show that $I_n = \dfrac{n-1}{n} I_{n-2}$, for $n = 2, 3, 4, \ldots$. **1**

(iii) Find the exact value of $I_9 \times I_{10}$. **2**

(b)



In the diagram above, triangle ABC is right-angled at B. Its circumcircle is drawn, with centre O. A point D is chosen on the circumcircle, then DE and DF are drawn perpendicular to AC and AB respectively. The point G is the intersection of DB and EF.

NOTE: You do not have to copy the diagram. It has been reproduced for you on a tear-off sheet at the end of the paper. Insert the tear-off sheet into your answer booklet.

(i) Explain why $ADEF$ is a cyclic quadrilateral. **1**

(ii) Let $\angle DAE = \theta$.
Prove that $\triangle FGB$ is isosceles. **2**

(iii) Prove that $ODEG$ is a cyclic quadrilateral. **2**

(iv) Deduce that OG is perpendicular to BD. **1**

Exam continues next page ...

i) $\angle AFD = \angle AED = 90^\circ$ (given)

$\therefore$ ADEF is a cyclic quadrilateral (equal angles at the circumference are subtended by the same arc)

ii) $\angle DAE = \angle DFE$ (angles at the circumference subtended by the same arc DE)

Similarly $\angle DAC = \angle DAE = \angle DBC$

$\therefore \angle EFB = 90 - \theta$ (adjacent complementary angles)

similarly $\angle FBD = 90 - \theta$

$\therefore \angle EFB = \angle FBD$

$\therefore \Delta FGB$ is isosceles (2 equal interior angles)

iii) let $\angle OAD = \theta$

$OA = OD$ (equal radii of a circle)

$\therefore \angle OAD = \angle ODA = \theta$ (angles opposite equal sides of a Δ are $=$)

$\therefore \angle EOD = 2\theta$ (exterior $\angle$ of a Δ is $=$ to the sum of the 2 opposite interior angles)

$\angle BGF = 2\theta$ (angle sum of a Δ)

$\therefore \angle EGD = 2\theta$ (vertically opposite angles)

$\therefore \angle EOD = \angle EGD = 2\theta$

$\therefore$ ~~GO~~ ODEG is a cyclic quadrilateral (equal angles at the circumference are subtended by the same arc).

iv) $\therefore \angle OGD = \angle OED = 90^\circ$ (angles at the circumference subtended by arc OD)

$\therefore OG \perp BD$